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Aggregate Set-utility Fusion for Multi-Demand – Multi-Supply Systems

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Abstract – *Microeconomic theory develops demand and supply curves to determine the market equilibrium for commodity exchange. The demand and supply functions are the result of consumers utilities and producers production functions for different product combinations. The interaction is a game-theoretic approach to determine the quantity and prices with which goods and services are traded. Economic theory works with the long run equilibrium concept, yet with constant alteration of information, decisions are made in the short run. People, with changing preferences, shift their fused-aggregate utility function for a set of preferences rather than a single commodity. The paper investigates a set-utility function based on a fused perception of the dynamic changes of the corporate supply and consumer demand curves for various products.*

1. Introduction

The interaction between supply and demand policies of households and corporations is dependent on prices and quantities [1,2]. The interaction between these variables model market events such as the clearing price in exchange. Analyzing these policies is difficult when people's preferences vary in time, substitutes and competing goods change, and the value of money is altered by other markets. For instance, if price information is coming from a variety of sources, it might have different reported ranges dependent on the source. However, these price resolutions can be fused to form a composite set of information which allows a consumer or a producer to make decisions on how to determine a fair price based on how much a corporation wants to produce and how much a consumer wants to spend. A corporation uses prices to alter the exchange quantity of goods, which imparts changes to people's spending behavior.

The purpose of the paper is to address the different resolutions of measurement microeconomic data that drives corporation's production policies and is similar to the macroeconomic model from Blasch [3]. This paper is organized in the following fashion. Section 2 presents the economic model for demand and supply functions and discusses time-delay errors that corrupt these measurements. Section 3 presents the multiresolution technique for fusing, propagating, and updating measured price states that result from dynamic quantity changes in supply and demand. Section 4 formulates the problem and section 5 presents simulated results. Finally, Section 6 discusses some concluding remarks.

2. Microeconomic Model

Microeconomic theory seeks to model the economy as a function of demand and supply functions. The *Demand Function (Q_d)*, is the relationship of quantity demanded to product prices and consumer income. The *Supply Function (Q_S)* is relationship of quantity supplied to production costs of wage rates and capital inputs [2]. The functional equilibrium determines the price of goods.

$$Q_d = Q_s \quad (1)$$

A dynamical equilibrium exists between prices and quantities and is cyclical between households and businesses through the goods and factors markets, shown in Figure 1.

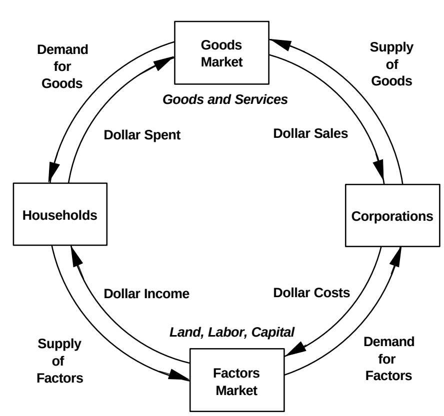

Figure 1. Exchange of Quantity and Prices [1].

The price-quantity model assumes that prices represent the value of goods. The goods market equilibrium shows a set of good's price and people's income from labor where:

$$Q_d = f(p_1, ..., p_n, m) \tag{2}$$

where *m* is the amount of consumers income [2]. From *utility theory*, income is the wealth constraint for quantity demanded,

$$m = Q_1 p_1 + \ldots + Q_n p_n \tag{3}$$

which shows that all decisions are based on a set of products a consumer can purchase.

Rearranging, we have:

$$Q_1 = \frac{m}{p_1} - \ldots - \frac{p_n}{p_1} Q_n \tag{4}$$

By determining the indifference point at which a consumer will equally value products, a marginal rate of substitution (MRS), (p_n/p_1), is determined for each good in a set.

A price consumption curve can be drawn for equilibrium sets of goods as one price changes, keeping other prices and income fixed. For each good, we can draw the inverse relationship between price and quantity demanded called the *law of demand* [1].

The quantity of goods supplied is a function of the corporation's production function. In the long run, all inputs are variable; however, in the short run, inputs of capital, K, and labor, L, are fixed. The relationship for capital and labor Q = f(K,L) is:

$$Q_s = \Delta L(MP_L) + \Delta K(MP_K) \tag{5}$$

where MP_L is the marginal product of labor, MP_K is the marginal product of capital, and $\Delta K/\Delta L = -MP_L/MP_K$ at zero output is termed the marginal rate of technical substitution (MRTS). The MRTS can also be related to the wage rate, *w*, and rental price of capital, *r*, by:

$$\frac{MP_L}{MP_K} = \frac{-\Delta K}{\Delta L} = \frac{w}{r} \tag{6}$$

Thus, we have a relation between the quantity supplied and the income consumers receive. The price equilibrium is shown as a relationship between quantity supplied and quantity demanded as shown in the Figure 2.

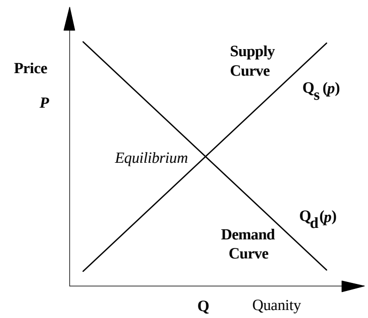

Figure 2. Price-Quanitity Equlibrium.

Using the models for quantity as a function of price, a state and measurement model is formed. Quantities and prices are variables and by inverting the demand function, we have 2 simultaneous equations:

$$\begin{bmatrix} \dot{Q}_{supply} \\ \dot{P}_{demand} \end{bmatrix} = \begin{bmatrix} C_s(wL,rk) \\ B_d(m,p_n) \end{bmatrix} \begin{bmatrix} Q \\ P \end{bmatrix} + \begin{bmatrix} w_s \\ w_d \end{bmatrix} \tag{7}$$

$$\begin{bmatrix} z_s \\ z_d \end{bmatrix} = \begin{bmatrix} H_s & H_d \end{bmatrix} \begin{bmatrix} Q \\ P \end{bmatrix} + \begin{bmatrix} n_s \\ n_p \end{bmatrix} \tag{8}$$

where $C(mL, rK)$ is the cost of the producer and $B(m, p)$ is the budget constraint of the consumer. By including uncertainty in the models, $v(t)$ and $w(t)$ are zero-mean mutually independent white Gaussian noise sequences with known covariances $Q(t)$ and $R(t)$, respectively.

The monitoring of economic variables is dependent on availability, time of measurement, and reporting confidence. If the reporting producer and consumer have time to fuse many perceived estimated values and quantities, the confidence is high, but requires delays in the updating of the information. The reporting time and confidence can be formulated as a multiresolution fusion problem, where multiple consumers and producers update knowledge of information at different time intervals.

3. Multi-Demand/Supply Relationships

The multiresolutional approach [4,5] propagates state values given sequential measurements. To develop the system equations for this approach, each point in time is expressed based upon the starting point of the block of time values. Figure 3 illustrates the decomposition and fusion that is described by the following equations. The basic state equation is:

$$\underline{x}_{k+1} = A_k \underline{x}_k + B_k \underline{w}_k \tag{9}$$

which may represent multiple demand-supply functions.

The second time point in time, based upon the *current state*, is expressed by

$$\begin{aligned} \underline{x}_{k+2} &= A_{k+1} \underline{x}_{k+1} + B_{k+1} \underline{w}_{k+1} \\ &= A_{k+1} A_k \underline{x}_k + A_{k+1} B_k \underline{w}_k + B_{k+1} \underline{w}_{k+1} \end{aligned} \tag{10}$$

The initial condition for the *first propagation time* state x and covariance P, a measure of uncertainty, may be expressed as

$$\hat{x}_{0|0}(k_N) = \begin{bmatrix} A \\ A \end{bmatrix} \underline{x}_0 , \tag{11}$$

$$P_{0|0}(k_N) = \begin{bmatrix} A \\ A \end{bmatrix} \underline{P}_0 \begin{bmatrix} A \\ A \end{bmatrix}^T + B_0 Q_0 B_0^T , \tag{12}$$

where $B_0 = \begin{bmatrix} B & 0 \\ A & B \end{bmatrix}$, and

$$Q_0 = diag \left\{ \begin{bmatrix} Q(0) & 0 \\ 0 & Q(0) \end{bmatrix} \begin{bmatrix} Q(1) & 0 \\ 0 & Q(1) \end{bmatrix} \right\}$$

The equations for a *blocked-time system* may be written as:

$$\underline{x}_{m+1} = \bar{A}_m \underline{x}_m + \bar{B}_m \bar{\underline{w}}_k , \tag{13}$$

where $\underline{x}_m = [\, \underline{x}_k^T , \underline{x}_{k+1}^T]^T$ and

$$A_m = diag\, [A_{k+1}, A_k]$$

Based upon the first observation, time k_4, the *estimate* is propagated at the highest resolution (N = 4):

$$\tilde{\underline{x}}(k_4) = A \, \hat{\underline{x}}, \tag{14}$$

$$\tilde{P}(k_4 + 1|k_4) = A_{k_4} P(k_4|k_4) A_{k_4}^T + B_{k_4} Q_{k_4} B_{k_4}^T \tag{15}$$

Using the measurement matrix:

$$z_{k_4} = H_{k_4} x(k_4) + v(k_4) \tag{16}$$

the *update* covariance is immediately computed:

$$x_{k_4+1|k_4+1} = \tilde{\underline{x}}(k_4) + K_{k_4}[z_{k_4} - H_{k_4} \tilde{\underline{x}}(k_4)]$$

$$P_{k_4+1|k_4+1} = [I - K_{k_4} H_{k_4}] \tilde{P}(k_4) \tag{17}$$

where K_{k_4} is the Kalman Gain.

Now, the generalized equations are derived using a wavelet approach to propagate Kalman-filtered updated states in time.

3.1 Discrete Wavelet Transform

For a given sequence of signals $x(i,n) \in L^2(\mathbb{Z})$, $n \in \mathbb{Z}$ at resolution level i, the *lower* resolution signal can be derived by:

$$x(i-1, n) = \sum_k h(2n-k) x(i,k) \tag{18}$$

The *added detail* is given by:

$$y(i-1, n) = \sum_k g(2n-k) x(i,k) \tag{19}$$

The original signal $x(i,k)$ can be recovered from two filtered and sub-sampled signals $x(i-1, n)$ and $y(i-1, n)$.

$$x(i,n) = \sum_k h(2k-n) x(i-1,k) + \sum_k g(2k-n) y(i-1,k) \tag{20}$$

The *lowpass* filter $h(n)$ is the impulse response of a Quadrature Mirror Filter (QMF) and $g(n)$ and $h(n)$ form a conjugate mirror filter pair:

$$g(L-1-n) = (-1)^n h(n) \tag{21}$$

where L is the filter length. The derivation here is similar to [4], with implementation coming from [5] where the *Daubechies' Filter* [6] is used for processing information at various resolutions. A more rigorous approach of wavelet filters can be found in Strang [7].

Consider a *finite* sequence of n-dimensional *random*

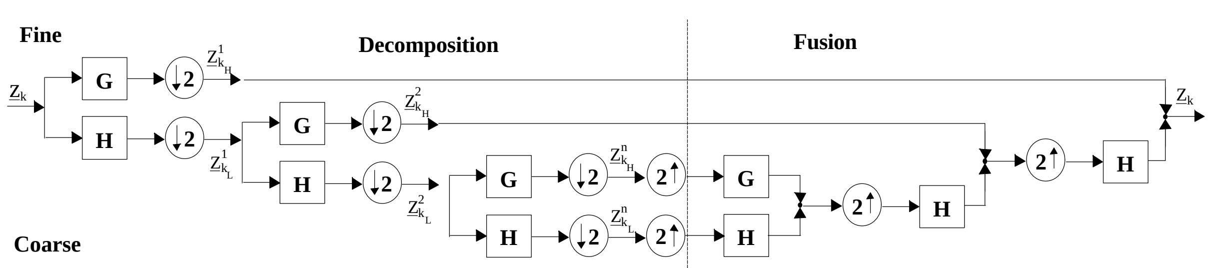

Figure 3. Control Flow for Distributed Multiresolutional Filtering.

vectors at resolution level i with a length of a data-block:

$$\underline{X}(k_i) = [\underline{x}^T(k_i), \underline{x}^T(k_i + 1),...,\underline{x}^T(k_i + 2^{(i-1)} - 1)]^T \quad (22)$$

To change $\underline{X}(k_i)$ to the form required by the wavelet transform, a linear transformation is introduced [5]:

$$\underline{X}'(k_i) = \mathbf{L}_i \, \underline{X}(k_i) \quad (23)$$

where L_i is a matrix of 1's and 0's which transforms the data order, but not the magnitude of the data.

The wavelet transform vector form is:

$$\underline{X}(k_{i-1}) = \mathbf{L}^T_{i-1} \bullet diag\{\mathbf{H}_{i-1}, \ldots, \mathbf{H}_{i-1}\} \bullet \mathbf{L}_i \bullet \underline{X}(k_i) \quad (24)$$
$$\underline{Y}(k_{i-1}) = \mathbf{L}^T_{i-1} \bullet diag\{\mathbf{G}_{i-1}, \ldots, \mathbf{G}_{i-1}\} \bullet \mathbf{L}_i \bullet \underline{Y}(k_i)$$

where $\mathbf{H}_{i-1}$ and $\mathbf{G}_{i-1}$ are scaling and wavelet operators. Similarly, mapping from level (i - 1) to level (i) can also be written as:

$$\underline{X}(k_i) = \mathbf{L}^T_i \bullet diag\{\mathbf{H}^T_{i-1}, \ldots, \mathbf{H}^T_{i-1}\} \bullet \mathbf{L}_{i-1} \bullet \underline{X}(k_{i-1})$$
$$+ \mathbf{L}^T_i \bullet diag\{\mathbf{G}^T_{i-1}, \ldots, \mathbf{G}^T_{i-1}\} \bullet \mathbf{L}_{i-1} \bullet \underline{Y}(k_{i-1}) \quad (25)$$

Since G_{i-1} is a *highpass* filter operator and the sequence $\underline{X}(k_i)$ is a noise driven one, $\underline{Y}(k_{i-1})$ is a sequence of "noise-like" signals. However, the sequence $\underline{Y}(k_{i-1})$ is *not white* and is correlated with $\underline{X}(k_{i-1})$ - *lowpass* filtered.

3.2 Distributed Multiresolution Filtering

The equations for the distributed multiresolution filtering are presented in [8]. The general methodology is performed by:

1. Propagating from m to m +1, where m is the money price -quantity (p, q) value.
2. Transmit (p, q)-1 estimate to (p, q)-2 update
3. Perform (p, q)-2 measurement updates
3a. Transmit (p, q)-1 predicted values to (p, q)-4,
4. Transmit (p, q)-1 updates to the (p, q)-4 site
5. Estimate fusion of (p, q)-1 and (p, q)-4 results
6. Propagate the (p, q)-4 update

Note that there is a time multiresolution fusion of market data at the equilibrium points and a spatial fusion of demand and supply curves which is similar to a multirate-multiresolutional filtering problem.

4. Problem Formulation

The system being investigated is an market model with four prices and quantities for a product. Since each consumer/producer has only partial information about the market (due to the uncertainties of data collection), it is naturally desired that four sources of measurements, from four observations, be fused to achieve a higher confidence about the state of the market.

Since the prior information about the market is nearly linear, the *dynamics* are approximated by the linear relationships plus a modeling error given by:

$$x(k+1) = x(k) + 1.5y(k) + w_x(k) \;, \; w_x(k) \sim N(0,\sigma)$$
$$y(k+1) = x(k) - 1.5\, y(k) + w_y(k) \;, \; w_y(k) \sim N(0,\sigma)$$

$$\text{or, } [\,\underline{x}_{k+1}] = [\,\Phi\,] \bullet [\underline{x}_k] + [\underline{w}_k] \,, \underline{w}_k \sim N(0,\mathbf{Q})$$

where k is time, the modeling error covariance matrix $\mathbf{Q}$ is given by $\mathbf{Q}$ = *diag* {10, 10}, and $w_x(k)$ and $w_y(k)$ are uncorrelated. The initial values are $\mathbf{P}_0$ = [$\mathbf{I}$] and $\underline{x}_{k_4}$ = $[300,0]^T$, assuming that the prices are in the range {200,400}. The *measuring process* for producers and consumers is described by the following measurement models, each of which is represented at their own timely and economic perspectives:

$$[\,\underline{z}^i_k] = [\mathbf{H}^i\,] \bullet [\underline{x}^i_k] + [\underline{v}^i_k] \,, \underline{v}^i_k \sim N(0, \mathbf{R}^i) \quad (26)$$

where the measurement matrices $\mathbf{H}_i$, i = 1,...,4 are identity matrices and the measurement error covariance matrices $\mathbf{R}_i$ are $\mathbf{R}_1$= *diag*{10,10}, $\mathbf{R}_2$ = *diag*{20,20}, $\mathbf{R}_3$ = *diag*{30,30}, and $\mathbf{R}_4$ = *diag*{40,40}, proportional to

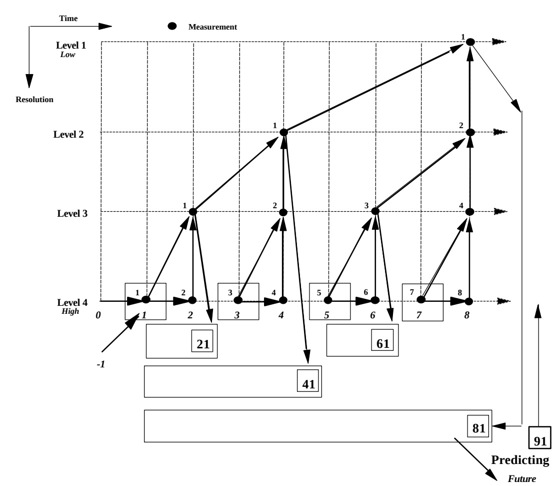

Figure 4. Real-time Multiresolution.

the resolution where timely measurements are assumed to have less accuracy than blocked updates.

Simulation runs are completed for 352 measurements, which is approximately a business year. The measurements are combined into 4, 2, and 1 measurements periods. Figure 4 shows how real-time values are propagated in time. Likewise, semi-real-time value updates are shown in Figure 5.

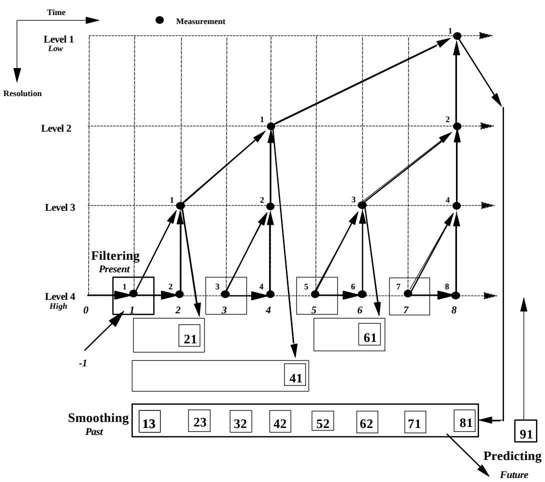

Figure 5. Semi-Real Time Multiresolution.

5. Simulation Results

A MATLAB program using the Daubechies' filter, the wavelet-multiresolution technique, simulates the multiresolution Kalman filter's performance for a set of time and quantity reports. Block time processing consists of measuring multirate states, processing the information at various resolutions, and fusing the results. In addition, the prediction function at the end of each time-block update predicts the time-associated next measurement. Level 4 is the real-time approach with 8 measurements used in the fusion process. Level 1, 2, and 3 are the semi-real-time approaches where measurements are processed, fused, and compared at various levels to the system (truth) model. Note that a *real-time* multiresolutional sensor fusion method is used to estimate the state equilibrium by fusing the information, sometimes from a single observer, since only the highest resolution is desired during the analysis.

5.1 Economic Measured Inputs

Input data is the result of measuring the market at different resolutions. Figures 5-8 show the four resolution of inputs, where it is assumed one demand/supply function update has highest resolution, but the largest variance.

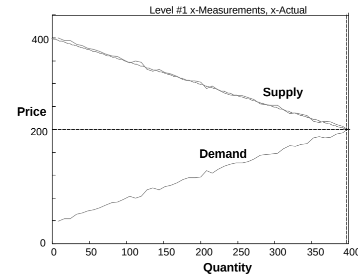

Figure 6. Spatio-Temporal Resolution Level 1.

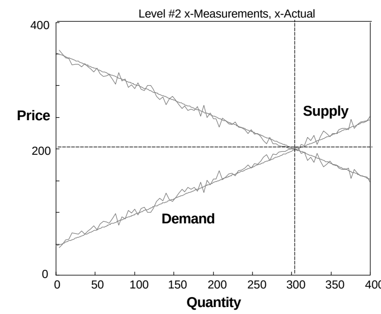

Figure 7. Spatio-Temporal Resolution Level 2.

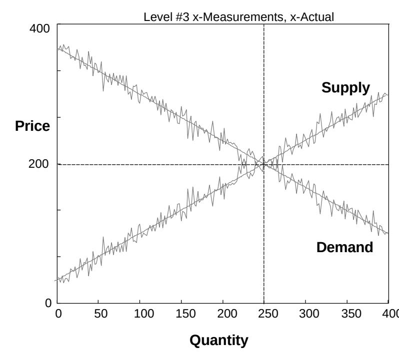

Figure 8. Spatio-Temporal Resolution Level 3.

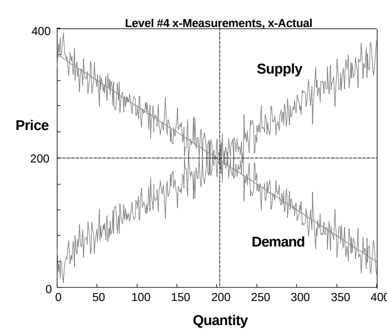

Figure 9. Spatio-Temporal Resolution Level 4.

5.2 Economic Estimated Outputs

For each set of value of a level corresponds to the consumer/producer resolution. By waiting, the consumer/producer would have a better estimated market value for variables in the demand/supply functions. From the fused result, we see that if we fuse curves and resolutions, we have a better estimate of aggregate prices and quantities.

6. Discussion and Conclusions

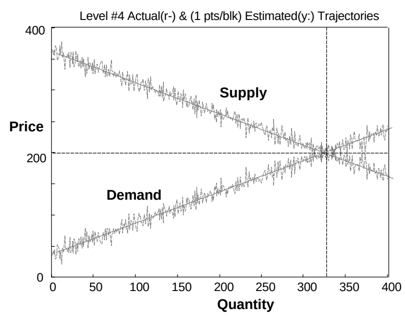

Figure 10. Fused Result at Highest Resolution.

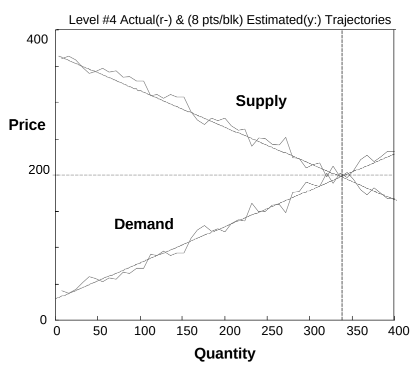

Figure 11. Fused Result at Coarsest Resolution.

The results show that estimation by the multiresolution technique allows for a variety of time fused updates dependent upon data variability and measurement confidence. Typically, measuring market data is the aggregate average perceived value. Since information available from different markets is reported at a variety of times, the methodology would be appropriate to incorporate data from a multiple set of observations. The difficulty with the analysis is that prices and quantities are typically observed values lagged in time, as shown in Figure 12, where the curve shifts to the right, but we only have the information from the measured curves, shown as dashed in Figure 12.

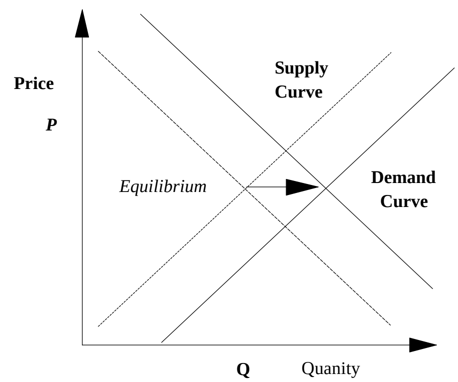

Figure 12. Policy Changes [1].

The multiresolution technique, used for sensor fusion models, is appropriate for assessing the time-delay updates associated with microeconomic system models. These results show that the model developed is applicable to updating consumers and producers with timely fused-estimates of variables for demand and supply functions.

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