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Decision Making in Multi-Fiscal and Multi-Monetary Policy Measurements

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Abstract *– This paper presents a multiresolution-economic model which mediates data uncertainty by providing levels of discernment for decision making. Standard monetary and fiscal policy models use national income and national output as input variables, perform a stability analysis by balancing the equations, and assess the interest rate of the economy. These models typically assume that data is accurately measured, timely, and available and drive interest-rate decisions. By exploring a multiresolution technique for daily and weekly policy updates, a data-fused macroeconomic model affords economists a real-time solution for policy strategy assessment.*

1. Introduction

The interaction between fiscal and monetary policies is dependent on national output, national income, and the interest rate [1]. The interaction between these variables model economic events such as the bond market's demand response to federal reserve alterations of the interest rate. Analyzing these policies is difficult when the measurement systems vary with time, accuracy, and validity of national data. For instance, if the information is coming from a variety of sources, it might have different reported confidence range updated daily or weekly. However, these data resolutions can be fused to form a composite set of information which allows the federal reserve to make decisions on how to keep the economy in equilibrium. The federal reserve uses the money supply to alter the federal interest rate, which imparts changes to people's spending and savings behavior.

The purpose of the paper is to address the different resolutions of measurement data that drives fiscal and monetary policies. This paper is organized in the following fashion. Section 2 presents the economic model for fiscal and monetary policies, a sample stability analysis, and discusses time-delay errors that corrupt these measurements. Section 3 presents the multiresolution technique for fusing, propagating, and updating measured states that result from dynamic movements of national income and national output. Section 4 formulates the problem and section 5 presents simulated results. Finally, Section 6 discusses some concluding remarks and section 7 details future research directions.

2. Macroeconomic Model

Macroeconomic theory seeks to model the economy as a function of the monetary and fiscal policies. The *monetary policy (LM)*, which is the relationship between money demand, interest rate, and national income, is controlled by a federal government's ability to print money. The *fiscal policy (IS)* is the interaction of the interest rate and the national income and national output [2]. The IS-LM model emphasizes the interaction between goods and assets markets, shown in Figure 1. The IS-LM model assumes that the national income affects people's spending which alters the national output and interest rates. High income raises the demand for money and interest rates.

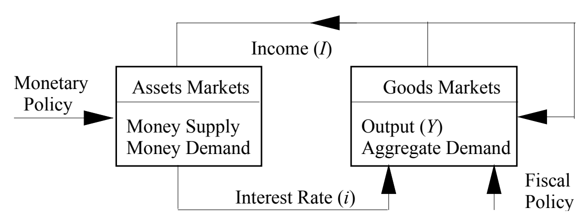

Figure 1. Fiscal and Monetary Policies [1].

However, high interest rates lower people's spending and adjust their income level. Income, spending, and interest rates are determined jointly by equilibrium in goods and assets markets. The goods market equilibrium schedule shows combinations of interest rates and the levels of output such that planned spending equals income [2]. The equilibrium income is:

$$Y_0 = \frac{A}{1 - c(1-t)} \tag{1}$$

where Y is the national income, A is autonomous spending, c is the propensity to consume out of income, t is the tax rate, and $1 - t$ is the money available after taxes. Investment spending is:

$$\Delta I = -bi \tag{2}$$

where i is the interest rate and b is the measurement of investment sensitivity to interest rates. Aggregate demand is the sum of spending which is equal to the national income:

$$AD = Y = A + cY - bi = \alpha\,(A - bi) \tag{3}$$

where $\alpha = 1/(1 - c)$. These factors are the *fiscal policy* $F(Y,i)$ which is discounted by people's saving and the government's spending.

To determine the *monetary policy*, we have to look at the money supply. Many countries, such as those in Latin America, have misused this policy in printing money to help finance political campaigns. The result of producing a substantial amount of money affects the fiscal policy and drives up interest rates. To relieve these countries of this dilemma, they had to peg their currency to that of a more stable country, such as the United States. Since monetary policy is crucial to world economic stability, it is imperative to have timely and accurate updates of the model [3].

The wealth budget constraint, where a person decides how to allocate money[1], implies that when the money market is in equilibrium, so is the bond market. Thus, federal government changes in the money supply affects the interest rate. The demand for money is the demand for real balances, since the public holds money for things it will buy. The demand for money, or real balances, is

$$MD = L = kY - hi = M/P \tag{4}$$

where k and h are sensitivity variables relating the money demand to income and interest rates, M is the stock of money and P is the price.

Using the models for demand, a state and measurement model is formed using the national output and interest rate as variables [1].

$$\begin{bmatrix} \dot{y}_{fiscal} \\ \dot{i}_{monetary} \end{bmatrix} = \begin{bmatrix} F(Y,i)-S(Y)+G \\ L(Y,i) - m_s \end{bmatrix} \begin{bmatrix} Y \\ i \end{bmatrix} + \begin{bmatrix} w_F \\ w_M \end{bmatrix} \tag{5}$$

$$\begin{bmatrix} z_f \\ z_{mr} \end{bmatrix} = \begin{bmatrix} H_f & H_m \end{bmatrix} \begin{bmatrix} y \\ i \end{bmatrix} + \begin{bmatrix} v_f \\ v_m \end{bmatrix} \tag{6}$$

where $S(y,t) - G$ is savings minus government spending. By including uncertainty in the models, $v(t)$ and $w(t)$ are zero-mean mutually independent white Gaussian noise sequences with known covariances $Q(t)$ and $R(t)$, respectively. Applications of simple control theory and filtering of macroeconomic systems can be found in [4,5].

The standard macroeconomic stability analysis of the two models is shown graphically in the figure below [2].

The monitoring of economic variables is dependent on availability, time of measurement,

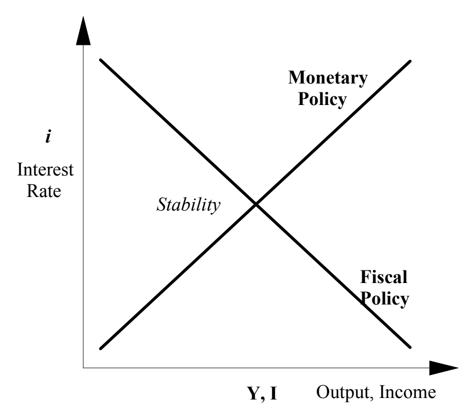

Figure 2. IS-LM Stability Analysis.

and reporting confidence. If the reporting center has time to integrate over many perceived estimated values, the confidence is high but causes delays in the updating of the information. The reporting time and confidence can be formulated as a multiresolution fusion problem, where multiple reporting centers update at different time intervals.

3. Multi-Fiscal/Monetary Policies

The multiresolutional approach [6,7] propagates state values given sequential measurements. To develop the system equations for this approach, each point in time is expressed based upon the starting point of the block of time values. The basic equation is:

$$\underline{x}_{k+1} = A_k \underline{x}_k + B_k \underline{w}_k \tag{7}$$

The second time point in the block, based upon the *current state*, is expressed by

$$\begin{aligned}\underline{x}_{k+2} &= A_{k+1} \underline{x}_{k+1} + B_{k+1} \underline{w}_{k+1} \\ &= A_{k+1} A_k \underline{x}_k + A_{k+1} B_k \underline{w}_k + B_{k+1} \underline{w}_{k+1}\end{aligned} \tag{8}$$

The initial condition for the *first propagation block* state x and covariance P, a measure of uncertainty, may be expressed as

$$\hat{x}_{0|0}(k_N) = \begin{bmatrix} A \\ A \end{bmatrix} \underline{x}_0 , \tag{9}$$

$$P_{0|0}(k_N) = \begin{bmatrix} A \\ A \end{bmatrix} \underline{P}_0 \begin{bmatrix} A \\ A \end{bmatrix}^T + B_0 Q_0 B_0^T, \tag{10}$$

where $B_0 = \begin{bmatrix} B & 0 \\ A & B \end{bmatrix}$, and

$$Q_0 = diag\left\{ \begin{bmatrix} Q(0) & 0 \\ 0 & Q(0) \end{bmatrix} \begin{bmatrix} Q(1) & 0 \\ 0 & Q(1) \end{bmatrix} \right\}$$

The equations for a *block system* may be written as:

$$\underline{x}_{m+1} = \bar{A}_m \underline{x}_m + \bar{B}_m \bar{\underline{w}}_k, \tag{11}$$

where $\underline{x}_m = [\, \underline{x}_k^T , \underline{x}_{k+1}^T]^T$ and

$$A_m = diag\,[A_{k+1}, A_k\,]$$

Based upon the first day of reporting, at time k_4, the *estimate* is propagated at the highest resolution (N = 4):

$$\tilde{\underline{x}}(k_4) = A\,\hat{\underline{x}}, \tag{12}$$

$$\tilde{P}(k_4 + 1|k_4) = A_{k_4} P(k_4|k_4) A_{k_4}^T + B_{k_4} Q_{k_4} B_{k_4}^T \tag{13}$$

Using the measurement matrix:

$$z_{k_4} = H_{k_4} x(k_4) + v(k_4) \tag{14}$$

the *update* covariance is immediately computed:

$$x_{k_4+1|k_4+1} = \tilde{\underline{x}}(k_4) + K_{k_4}[z_{k_4} - H_{k_4} \tilde{\underline{x}}(k_4)]$$

$$P_{k_4+1|k_4+1} = [I - K_{k_4} H_{k_4}]\, \tilde{P}(k_4) \tag{15}$$

where K_{k_4} is the Kalman Gain.

Now, the generalized equations are derived using a wavelet approach to propagate Kalman-filtered updated states in time.

3.1 Discrete Wavelet Transform

For a given sequence of signals $x(i,n) \in L^2(Z), n \in Z$ at resolution level i, the *lower* resolution signal can be derived by:

$$x(i-1, n) = \sum_k h(2n-k) x(i,k) \tag{16}$$

The *added detail* is given by:

$$y(i-1, n) = \sum_k g(2n-k) x(i,k) \tag{17}$$

The original signal $x(i,k)$ can be recovered from two filtered and sub-sampled signals $x(i-1, n)$ and $y(i-1, n)$.

$$x(i,n) = \sum_k h(2k-n) x(i-1,k) + \sum_k g(2k-n) y(i-1,k) \tag{18}$$

The *lowpass* filter $h(n)$ is the impulse response of a Quadrature Mirror Filter (QMF) and $g(n)$ and $h(n)$ form a conjugate mirror filter pair:

$$g(L-1-n) = (-1)^n h(n) \tag{19}$$

where L is the filter length. The derivation here is similar to [6], with implementation coming from [7] where the *Daubechies' Filter* [8] is used for processing information at various resolutions. A more rigorous approach of wavelet filters can be found in Strang [9].

Consider a *finite* sequence of n-dimensional *random vectors* at resolution level i with a length of a data-block:

$$\underline{X}(k_i) = [\underline{x}^T(k_i), \underline{x}^T(k_i+1), \ldots, \underline{x}^T(k_i + 2^{(i-1)} - 1)]^T \tag{20}$$

To change $\underline{X}(k_i)$ to the form required by the wavelet transform, a linear transformation is introduced:

$$\underline{X}'(k_i) = \mathbf{L}_i\, \underline{X}(k_i) \tag{21}$$

where L_i is matrix of 1's and 0's which transforms the order of the data, but not the magnitude of the data.

The vector form of the wavelet transform can be derived:

$$\underline{X}(k_{i\text{-}1}) = \mathbf{L}^T_{i\text{-}1} \bullet diag\{\mathbf{H}_{i\text{-}1}, \ldots, \mathbf{H}_{i\text{-}1}\} \bullet \mathbf{L}_i \bullet \underline{X}(k_i) \tag{22}$$
$$\underline{Y}(k_{i\text{-}1}) = \mathbf{L}^T_{i\,\text{-}\,1} \bullet diag\{\mathbf{G}_{i\text{-}1}, \ldots, \mathbf{G}_{i\text{-}1}\} \bullet \mathbf{L}_i \bullet \underline{Y}(k_i)$$

where $\mathbf{H}_{i\,\text{-}1}$ and $\mathbf{G}_{i\,\text{-}1}$ are scaling and wavelet operators. Similarly, mapping from level (i - 1) to level (i) can also be written as:

$$\underline{X}(k_i) = \mathbf{L}^T_i \bullet diag\{\mathbf{H}^T_{i\,\text{-}1}, \ldots, \mathbf{H}^T_{i\,\text{-}1}\} \bullet \mathbf{L}_{i\,\text{-}\,1} \bullet \underline{X}(k_{i\,\text{-}\,1})$$
$$+ \mathbf{L}^T_i \bullet diag\{\mathbf{G}^T_{i\,\text{-}1}, \ldots, \mathbf{G}^T_{i\,\text{-}1}\} \bullet \mathbf{L}_{i\,\text{-}\,1} \bullet \underline{Y}(k_{i\,\text{-}\,1}) \tag{23}$$

Since $G_{i\,\text{-}1}$ is a *highpass* filter operator and the sequence $\underline{X}(k_i)$ is a noise driven one, $\underline{Y}(k_{i\,\text{-}\,1})$ is a sequence of "noise-like" signals. However, the sequence $\underline{Y}(k_{i\,\text{-}\,1})$ is *not white* and is correlated with $\underline{X}(k_{i\,\text{-}\,1})$ - *lowpass* filtered.

3.2 Distributed Multiresolution Filtering

The equations for the distributed multiresolution filtering are presented in [10] and are shown in the figure below. The general methodology is performed by:

1. Propagating from time m to m +1, where m is the weekly value, which consists of daily updates.
2. Transmit weekly estimate to daily update
3. Perform daily measurement updates

3a. Transmit daily predicted values for the next week, such as just Monday's results.

4. Transmit daily updates to the weekly site
5. Estimate the fusion of daily and weekly results
6. Propagate the weekly update

Note that there is a daily vs. weekly time multiresolution flow of economic data at the various time and confidence resolution levels.

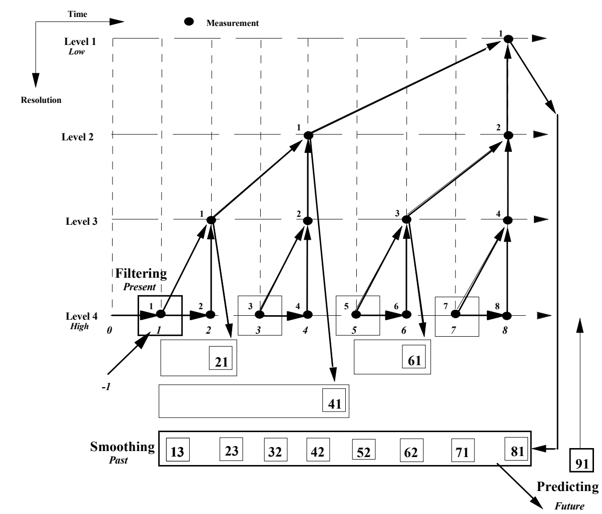

Figure 5. Semi-Weekly Multiresolution.

4. Problem Formulation

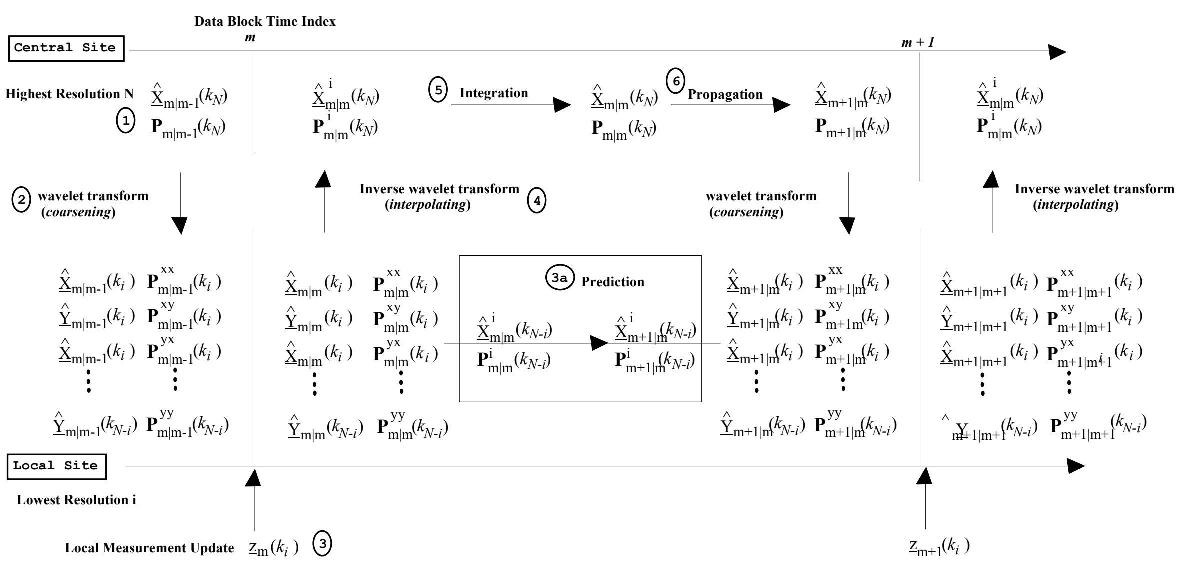

Figure 3. Control Chart for Distributed Multiresolutional Filtering and Block-Type Prediction.

The system being investigated is an economic model with four financial centers measuring the state of the economy. Since each center provides only partial information about the economy (due to the uncertainties of data collection and confidence level), it is naturally desired that four sources of the measurements, from four observing centers, be fused to achieve a higher confidence about the state of the economy.

Since the prior information about the economy is nearly linear, the *dynamics* are approximated by the linear relationships plus a modeling error given by:

$$x(k+1) = x(k) + 1.5y(k) + w_x(k) \ , \ w_x(k) \sim N(0,\sigma)$$
$$y(k+1) = x(k) - 1.5\, y(k) + w_y(k) \ , \ w_y(k) \sim N(0,\sigma)$$

or, $[\, \underline{x}_{k+1}] = [\, \Phi \,] \bullet [\underline{x}_k] + [\underline{w}_k] \, , \underline{w}_k \sim N(0,\mathbf{Q})$

where k is time, the modeling error covariance matrix $\mathbf{Q}$ is given by $\mathbf{Q}$ = *diag* {1.0 , 1.0}, and $w_x(k)$ and $w_y(k)$ are uncorrelated. The initial values are $\mathbf{P}_0 = [\mathbf{I}]$ and $\underline{x}_{k_4} = [10,0]^T$, assuming that the interest rate falls between 2 and 10 percent [3]. The *measuring processes* of the four resolution centers is described by the following measurement models, each of which is represented at their own local or daily economic perspectives:

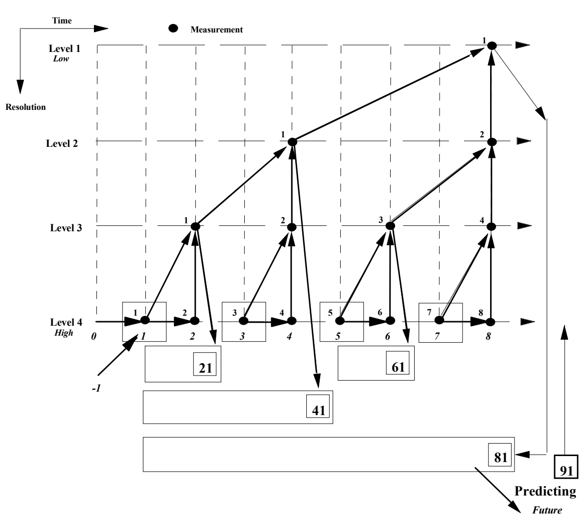

Figure 4. Daily Multiresolution.

$$[\, \underline{z}^i_k] = [\mathbf{H}^i\,] \bullet [\underline{x}^i_k] + [\underline{v}^i_k] \, , \underline{v}^i_k \sim N(0, \mathbf{R}^i) \qquad (24)$$

where the measurement matrices $\mathbf{H}_i$, i = 1,...,4 are identity matrices and the measurement error covariance matrices $\mathbf{R}_i$ are $\mathbf{R}_1$= *diag*{1.0,1.0}, $\mathbf{R}_2$ = *diag*{2.0,2.0}, $\mathbf{R}_3$ = *diag*{3.0,3.0}, and $\mathbf{R}_4$ = *diag*{4.0,4.0}, proportional to the resolution where daily measurements are assumed to have less accuracy than weekly updates.

The simulation runs are completed for 352 measurements, which is approximately a business year. The measurements are combined into four, two, and one measurements per week. Figure 4 shows how daily numbers are propagated in time. Likewise, the weekly updates, or partial weekly updates are shown in Figure 5.

5. Simulation Results

A MATLAB program using the Daubechies' filter, the wavelet-multiresolution technique, simulates the multiresolution Kalman filter's performance for a set of time processing. The time-block (a week) processing consists of measuring the daily, semiannually, or weekly state, processing the information at various resolutions, and fusing the results. In addition, the prediction function at the end of each time-block update predicts the time-associated next measurement. Level 4 is the daily approach with eight measurements per week used in the fusion process. Level 1, 2, and 3 are the semi-weekly approaches where measurements are processed, fused, and compared at various levels to the system (truth) model. Note that a *daily* multiresolutional sensor fusion method is used to estimate the state of the economy by fusing the information, sometimes from a single reporting center, since only the highest resolution is desired during the analysis.

For comparison purposes, the normalized error is defined by:

$$\varepsilon(k) = (\underline{x}_k - \hat{x}_{k|k})^T \, P^{-1}_{k|k} \, (\underline{x}_k - \hat{x}_{k|k}) \qquad (25)$$

The normalized error is used to compare each individual center to that of the time-fused result which occurs at the end of the week. To highlight the difference between the semi-

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weekly and daily approaches, the normalized errors show the differences.

5.1 Economic Measured Inputs

Input data is the result of measuring the economy at different resolutions. The figure below shows the four resolution of inputs, where it is assumed daily update has highest resolution, but the largest variance.

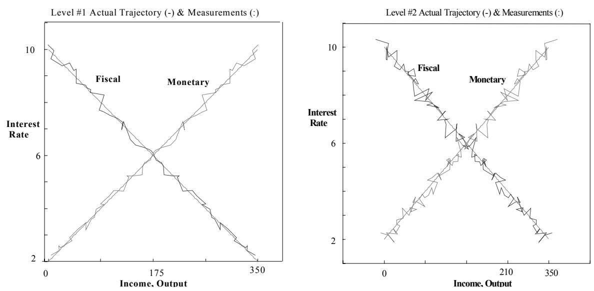

Figure 6. Weekly Resolutions – Level 1 and 2.

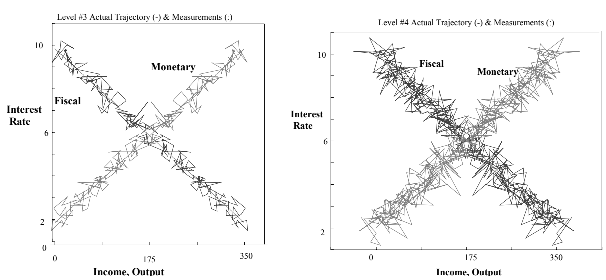

Figure 7. Daily Resolutions – Level 3 and 4.

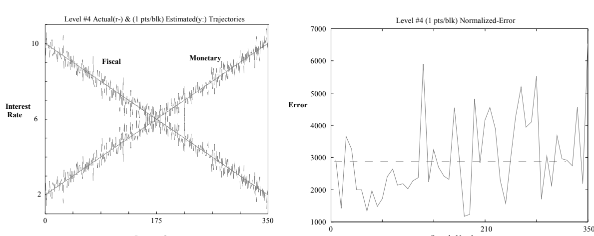

Figure 8. Real Time Daily Estimated Values.

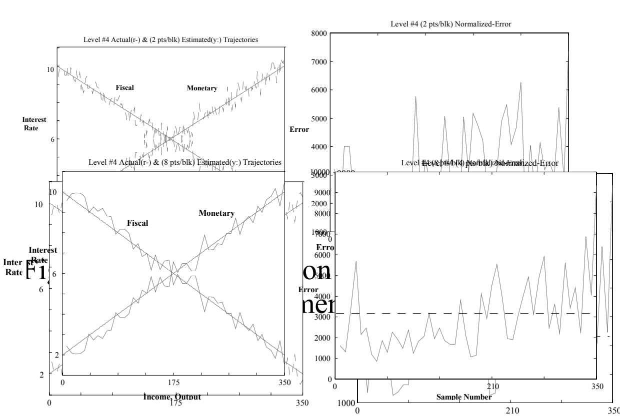

Figure 10. Semi-resolution Estimated Values. (2 Measurements/Week)

5.2 Economic Estimated Outputs

For each set of input data, the estimated output value is plotted. It is important to note that the value of a level corresponds to how many days an economist would have to wait for the estimated economic data. By waiting approximately a week, or 8 days, the economist would have a better estimated value for variables in monetary or fiscal policies. To assess the continuous model, the error was computed for the four approaches.

Table 1. Estimated Trace and Error Values

Msrmnts/week	MSE
1 Real time	2980
2	3260
4	3410
8 Semi-Realtime	3580

From these results, we see that if we wait till the end of the week, the uncertainty in the measurement increases, but only slightly. Thus, using the multiresolution approach would allow for daily-fused measurements for decision making.

6. Discussion

The results show that estimation by the multiresolution technique allows for a variety of time updates dependent upon data variability and measurement confidence. Typically, measuring economic data is the average perceived value that the economy experiences. Since information available from financial organizations is reported at a variety of times, the methodology would be appropriate to incorporate data from a multiple set of reporting centers. The difficulty with the analysis is that we only consider the inputs.

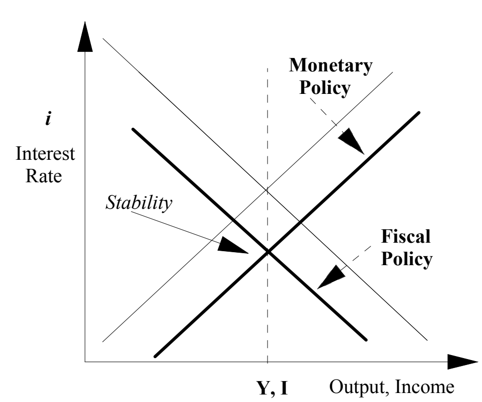

Figure 12. Policy Changes [1].

Many economists use the IS-LM model to predict what actions the federal government should take. For instance, if the federal reserve wants to displace the monetary policy, then the fiscal policy reactions would result to bring the interest rate down, assuming national income and output do not change, shown in Figure 12. Hence, with the measurement uncertainty, when should people act upon this information to determine the spending levels? If the federal government waits till the end of the week, then the policy would more accurately reflect the position of the economy. By using the multiresolution approach, the federal reserve could update their estimate with daily numbers before making a decision.

The balance of payments, Figure 13, are dependent upon these policy values and dependent upon the measured economic variables provided to users. The balance of payments is considered in equilibrium under a fixed-exchange rate when there is an equality of international payments with international receipts. However, domestic interest rates alter the exchange rates between countries. The decision-dependent output effects of international exchange rate changes can be computed to determine the domestic state of inputs. Aggregate autonomous spending, $A = f(Y)$ and the exchange rate $e = f(I)$ are related to the changes in the macroeconomic policy [2].

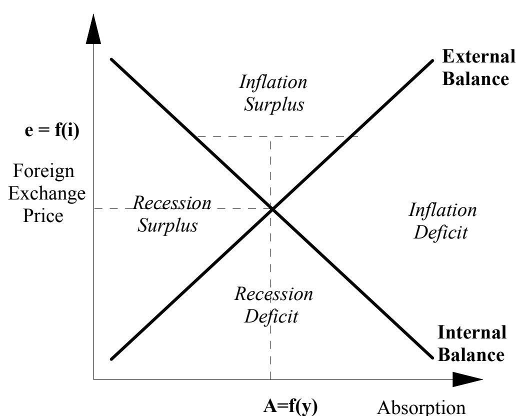

Figure 13. The Balance of Payments [2].

The results of the input and output actions will be explored in further papers and compared to actual data. Macroeconomic policy changes resulting from input and output movements of the measured economic data can then aid decision makers with dynamic exchange rates. For example, Evans calculates international integration of consumption and growth [11].

7. Conclusions

The multiresolution technique, used for sensor fusion models, is appropriate for assessing the time-delay updates associated with macroeconomic system models. These results show that the model developed is applicable to updating economists with daily and weekly fused-estimate of variables for fiscal and monetary policy decision making. Future research will focus on interactive global monetary and fiscal policies.

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